If for $\epsilon = 0.01$ the system were designed to attain $D = 30\Omega$, then one would have $c_1A_2 + c_2A_1 = 2DA_1A_2[2 - 0.01 \times 299] < 0$. Hence, a 1% error in this case would give a negative coefficient of $p^3$ and an unstable system.


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# Exhaustion of Geomagnetically Trapped Radiation


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THE amount of matter which must be injected into belts of geomagnetically trapped radiation in order to lower the radiation levels has been estimated. Two mechanisms are available for this purpose: the elastic scattering of particles into paths which will intersect the atmosphere before mirroring, and the removal of the particles' energy by ionization and excitation (i.e., the normal stopping power of matter).

The latter is easier to estimate. A particle of velocity $v$ (cm/sec) has a path length of $m$(g/cm$^2$) in matter that is relatively independent of the stopping material. If a time $t$ (sec) is allowed to stop it, the density of matter in the radiation belt must be $m/vt$ (g/cm$^3$). For a belt of volume $V$ (cm$^3$), one therefore must orbit a total mass $M = Vm/vt$ (g).

For the artificial radiation belt produced by recent nuclear tests, one may assume that the main component is electrons of approximately 1 Mev, so that $m$ can be taken as 0.5 g/cm$^2$ and $v$ as $3 \times 10^{10}$ cm/sec. The belt volume is about $10^{26}$ cm$^3$, and one obtains $Mt = 1.7 \times 10^{15}$ g-sec or about 50 ton-yr.

For the protons of the inner Van Allen belt, $m$ and $V$ are slightly larger and $v$ is slightly less, so that $Mt$ is of the order of 300 ton-yr. Here, however, there is an upper limit on $t$ which is given by the average natural residence time of the protons, which so far is unknown. The persistence of the artificial belt in a region of low natural radiation intensity suggests that residence times may be much longer and natural injection rates much lower than hitherto has been supposed. If this is so, a permanent reduction of radiation levels could be achieved in due course by the introduction of feasible amounts of matter. The reduction would proceed until particle concentrations are so low that the removal rate (natural and artificial) equals the arrival rate.

For the outer Van Allen belt, the values of $m$ and $v$ are comparable to those for the artificial belt, but $V$ is considerably greater, so that $Mt$ here is also of the order of 300 ton-yr. Again the maximum value of $t$ is unknown.

In order to scatter trapped particles out of the belts, they must be deflected into a narrow cone along the magnetic lines of force. It is assumed that it would require at least 10 collisions of more than 10° each to effect this for the average belt particle. The number of atoms per cubic centimeter, $N$, which is required to produce this scattering in $t$ sec is given by $N\sigma vt = 10$, where $\sigma$ is the cross section for elastic scattering of the belt particles through an angle of 10° or more. The cross section can be calculated from the Coulomb scattering law.[1] For 1 Mev electrons scattered off a material with an average atomic number of 3 (e.g., a hydrocarbon), one obtains $\sigma = 2 \times 10^{-23}$ cm$^2$. This gives $Nt = 1.7 \times 10^{14}$ sec $\times$ particles/cm$^3$ or, taking a belt volume of $10^{26}$ cm$^3$ and an average atomic weight of 5, $Mt = 450$ ton-yr. This is therefore a less efficient exhaustion mechanism than energy removal.

The cross section for elastic scattering is inversely proportional to the square of the particle kinetic energy. The elastic scattering mechanism is therefore even less efficient for the highly energetic particles of the inner Van Allen belt, when compared to the energy removal mechanism which is most efficient for heavy particles. In the outer Van Allen belt, the ratio of efficiencies for the two mechanisms is comparable to that in the artificial belt.

It should be noted that, because of the strong energy dependence of $\sigma$, the relative efficiency of the scattering process increases as the particle energy goes down. The two mechanisms therefore reinforce each other. Some consequences of these calculations are as follows.

It does not seem desirable or quite possible at present to orbit matter with the exclusive or primary purpose of removing the artificial or natural radiation belts. Objects already in orbit are performing this function rather slowly and inefficiently, since their configuration is not optimal for this purpose. With the future establishment of space stations on orbits intersecting the belts, it may become very desirable to sweep the belts out in the manner indicated. The space stations themselves would not be good for this purpose because of the personnel radiation doses that would be absorbed from bremsstrahlung.

The choice of material for a sweepout program is not particularly critical, but the physical state is quite important. A gas would dissipate too rapidly, whereas solid particles in the micron to centimeter size range would increase unduly the micrometeorite hazard. Large chunks of materials are inefficient as particle exhaustors because most paths through them are longer than the distance required to stop a particle. This leaves two efficient forms of matter distribution: as a colloid or as thin sheets. Optical scattering effects of the colloidal material may be objectionable for astronomical observations. The authors consider that the best way to orbit material for belt exhaustion would be as thin sheets of solid material. Aluminum would be very suitable because it is notably stable to ionizing radiation. The interference with astronomical observations would be minimal; 10 tons of 0.05-g/cm$^2$ sheet would have a maximum area of 20,000 m$^2$, or $10^{-4}$ sterad at 500 km.

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